Implicit Differentiation and the Second Derivative

Calculate y'' using implicit differentiation; simplify as much as possible.

$$x^2 + 4y^2 = 1$$

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$$x^2 + 4y^2 = 1$$

$$x^2 = 1 - 4y^2$$

$$\frac{d}{dx} x^2 = \frac{d}{dx}(1 - 4y^2)$$

$$2x = 0 - 8y \frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{1}{4} \frac{x}{y}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{4}\left[\frac{1 \cdot y - x \cdot \frac{dy}{dx}}{y^2}\right]$$

$$= -\frac{1}{4}\left(\frac{y + \frac{1}{4}\frac{x^2}{y}}{y^2}\right)$$

$$= -\frac{1}{4}\left(\frac{\frac{4y^2 + x^2}{4y}}{y^2}\right)$$

$$= -\frac{1}{4}\left(\frac{1}{4y^3}\right)$$

$$= -\frac{1}{16y^3}$$

$$y^2 = \frac{1 - x^2}{4}$$

$$y = \pm \frac{\sqrt{1 - x^2}}{2}$$

$$-\frac{1}{16y^3}$$

$$= -\frac{1}{4y(1 - x^2)}$$

$$= \frac{1}{\mp 2\sqrt{1 - x^2}(1 - x^2)}$$

$$= \mp \frac{1}{2(1 - x^2)^{3/2}}$$